

# St SimulationsPlus

#### Automated covariate selection: SAMBA and COSSAC algorithms

Claude Magnard

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NASDAQ: SLP

#### **Covariate selection**



#### (6 parameters x 6 covariates)<sup>2</sup> > 1000 possible covariate models



# **Covariate model building strategies**

SCM (stepwise covariate modeling) Available in PsN and Monolix

#### FREM (full random effects model) Available in PsN

COSSAC Available in Monolix LASSO Available in PsN

#### SCM+ Available in PsN

SAMBA Available in Monolix

WAM (Wald Approximation Method) Machine Learning (e.g random forest, neural networks)



# **Covariate model building strategies**

Most commonly used but requires many runs...

SCM (stepwise covariate modeling) Available in PsN and Monolix

#### FREM (full random effects model) Available in PsN

COSSAC Available in Monolix LASSO Available in PsN SCM+ Available in PsN

SAMBA Available in Monolix

Machine Learning (e.g random forest, neural networks)



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WAM (Wald Approximation Method)

# **SCM procedure**

#### 4 parameters: Tlag, ka, V, Cl and 3 covariates: Age, Weight, Sex

- Test all possible covariate-parameter relationships at each step
- Keep the one that improves LL the most
- Forward until no further addition, then backward

# runs ≈ #par x #cov x #relations

takes long to run...



# **Covariate model building strategies**

Most commonly used but requires many runs...

SCM (stepwise covariate modeling) Available in PsN and Monolix FREM (full random effects model) Available in PsN

COSSAC Avral *et al*, 2021

Available in Monolix

LASSO Available in PsN SCM+ Available in PsN

SAMBA

Prague & Lavielle, 2022

Available in Monolix

WAM (Wald Approximation Method) Machine Learning (e.g random forest, neural networks)





#### COSSAC

#### **CO**nditional **S**ampling use for **S**tepwise **A**pproach based on **C**orrelation tests

Ayral (Cellière), G., Si Abdallah, J. F., Magnard, C. & Chauvin, J. A novel method based on unbiased correlations tests for covariate selection in nonlinear mixed effects models: The COSSAC approach. *CPT Pharmacometrics Syst. Pharmacol.* **10**, 318–329 (2021).

## **COSSAC** Key idea

Using samples from the conditional distributions allows to reliably detect correlations between random effects and covariates





## **COSSAC** Key idea

eta ka

eta\_Cl

0.32

0.35

1.83

2.08

7.78e-2

4.65e-2

The p-values of the correlation tests can be used to select which covariates to try instead of trying all

Pearson's correlation test and/or ANOVA

	COEFF	STATISTICS	P-VALUE		COEFF	STATISTICS	P-VALUE
sex		1.14	2.95e-1	sex		0.036	8.5e-1
age	0.075	0.41	6.84e-1	age	0.0043	0.024	9.81e-1
wt	0.0013	0.0074	9.94e-1	wt	0.24	1.33	1.94e-1

These p-values are unbiased when using samples from the conditional distribution.

Low p-value = significant correlation between samples from conditional distribution and covariates



STATISTICS	P-VALUE		COEFF	STATISTICS	P-VALUE
18.59	1.61e-4	sex		0.38	5.43e-1

age

wt

eta\_Tlag

eta\_V

COEFF

-0.039

0.75

-0.21

6.24

8.32e-1

7.22e-7

sex

age

wt

# **COSSAC procedure: overview**

- 1. Calculate the p-value of the correlation test for each parameter-covariate pair in the current model
- 2. Forward step:
  - Add the most promising (lowest p-value < 0.3) covariate-parameter relationship.</p>
  - If the likelihood improves enough, keep the covariate and add another relationship on top
- 3. Backward step:
  - Remove the least significant (highest p-value > 0.01) already included relationship.
  - If the likelihood worsens too much, put covariate back in the model
- 4. Alternate forward and backward steps

#### → Covariates are added one by one (as in SCM, different from SAMBA)

Ayral (Cellière), G., Si Abdallah, J. F., Magnard, C. & Chauvin, J. A novel method based on unbiased correlations tests for covariate selection in nonlinear mixed effects models: The COSSAC approach. *CPT Pharmacometrics Syst. Pharmacol.* **10**, 318–329 (2021).



















#### 4 parameters: Tlag, ka, V, Cl and 3 covariates: Age, Weight, Sex



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## **Number of runs**

#### SCM: 43 runs # runs ≈ #par x #cov x #relations



#### COSSAC: 7 runs # runs ≈ #par x #cov



Ayral (Cellière), G. et al, CPT Pharmacometrics Syst. Pharmacol. (2021)



## **COSSAC: comparison to SCM on real datasets**

				COSS	COSSAC					
Data set	Characteristics	Parameters ( <i>italic:</i> no variability)	Covariates	No. runs	Final model	No. runs	Final model	ΔLL	ΔBICc	Ratio # runs
Remifentanil PK	Linear PK - SD - dense 65 indiv - 1992 obs	6 - CI, VI, Q2, V2, Q3, V3	6 - SEX, logAGE, logBSA, logHT, logLBM, logWT	13	SEX – V3 logAGE - Cl, Q2, V2, V3 logBSA - Cl logLBM – V1, V2	295	logAGE – Cl, Q2, <b>Q3</b> , V2, V3 logBSA - Cl logHT – <b>V2</b> logLBM – V1	-3.8	0.4	22.7
Theophylline PK	Linear PK - SD - dense 12 indiv - 120 obs	3 - ka, V, Cl	2 - SEX, logWT	6	None	7	None	Identic	al	1.2
Verapamil PK	Linear PK - SD - dense 22 indiv - 330 obs	6 - Tlag, ka, Cl, V1, Q, V2	7 - SEX, RACE, logAGE, logHT, logWT, logDIABP, logSYSBP	34	SEX - Cl, V1, ka logAGE - ka logWT - Q, V2	241	SEX - Cl, ka logAGE - ka logWT - Q, V2	-2.6	0.5	7.1
GBR12909 PK	Linear PK - MD - dense 12 indiv - 232 obs	5 - ka, V, k, k12, k21	2 - SEX, logWT	5	logWT - ka	20	logWT - ka	Identic	al	4
Quinidine PK	Linear PK - SD - dense 21 indiv - 315 obs	6 - Tlag, ka, Cl, V1, Q, V2	7 - SEX, RACE, logAGE, logHT, logWT, logDIABP, logSYSBP	20	SEX - CI, V1	124	SEX - CI, V1	Identic	al	6.2
Quinidine sparse PK	Linear PK - MD - sparse 136 indiv - 361 obs	3 - ka, V, Cl	7 - RACE, HEART, ETHANOL, SMOKE, logAGE, logHT, logWT	11	None	22	None	Identic	al	2
Tobramycin sp. PK	Linear PK - MD - sparse 97 indiv - 322 obs	2 - V, Cl	4 - SEX, logAGE, logCLCR, logWT	7	logCLCR - Cl logWT - V	22	logCLCR - Cl logWT - V	Identic	al	3.1
Cisplatine PK	Linear PK - MD - dense 23 indiv - 524 obs	6 - Cl, V1, Q2, V2, <i>Q3, V3</i>	5 - SEX, logAGE, logBSA, logHT, logWT	16	logBSA - V1	60	logBSA - V1	Identic	al	3.8
Theophylline ER PK	Linear PK - SD - dense 18 indiv - 362 obs	7 - ka1, ka2, F1, Tlag1, diffTlag2, V, Cl	3 - logAGE, logHT, logWT	17	logWT - Tlag1, V	61	logAGE – <b>ka2</b> logWT – Tlag1	0.5	0.5	3.6
IgG1 mAb PK	TMDD PK - SD - dense 28 indiv - 263 obs	7 - V, kint, kon, R0, Cl, Q, V2	2 - RA, logWT	13	RA - Cl, V, kon logWT - V	63	RA - Cl, V, kon logWT - V	Identic	al	4.8
Remifentanil seqPD	PD - SD - dense 61 indiv - 3989 obs	5 - ke0, E0, I <sub>max</sub> , IC <sub>50</sub> , gam (indiv. PK param fixed)	6 - SEX, logAGE, logBSA, logHT, logLBM, logWT	29	SEX - gam logAGE - IC <sub>50</sub> , gam, ke0 logHT - gam	194	SEX - gam logAGE - <b>E0</b> , IC <sub>50</sub> , gam, ke0 logHT - gam	8.4	4.3	6.7

Ayral (Cellière), G. et al, CPT Pharmacometrics Syst. Pharmacol. (2021)

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## **COSSAC: comparison to SCM on real datasets**

				COSS	COSSAC					
Data set	Characteristics	Parameters ( <i>italic:</i> no variability)	Covariates	No. runs	Final model	No. runs	Final model	ΔLL	ΔBICc	Ratio # runs
Dofetilide PK/PD	Joint PK/PD - SD - dense 22 indiv - 328x2 obs	8 - Tlag, ka, Cl, V1, Q, V2, intercept, slope	7 - SEX, RACE, logAGE, logHT, logWT, logDIABP, logSYSBP	60	RACE - intercept, ka logWT - V1	220	RACE - <i>intercept</i> logSYSBP - <i>intercept</i> logWT - V1	0.5	0.2	3.7
MIDD (ASCPT Gran Prix)	Joint model parent/ metab/urine/PD 176 indiv – 2664 + 2723+ 147 + 2600 obs	9 - ka, V, Cl, Clr, Clm, Vm, R0, kdeg, IC <sub>50</sub>	8 - SEX, ESRD, logAGE, logHT, logWT, logALB, nDiseases, nDrugs	20	ESRD - Cl logAGE - Cl logALB - Cl, Clr, V, ka logWT - Clr	421	logAGE - Cl logALB - Cl, V, ka nDiseases - k <sub>deg</sub>	-40	-29	21.1
Warfarin PK/ PD	Joint PK/PD - SD - dense 32 indiv - 247 + 232 obs	8 - Tlag, ka, V, Cl, R0, kout, I <sub>max</sub> , IC <sub>50</sub>	3 - SEX, logAGE, logWT	11	logWT - V	48	logWT - V	Identic	al	4.4
Cholesterol	Disease progression 200 indiv - 1044 obs	2 - Chol0, slope	2 - SEX, logAGE	5	logAGE - Chol0, slope SEX - slope	12	logAGE - Chol0, slope SEX - slope	Identic	al	2.4
Alzheimer	Disease - count data 896 indiv - 3707 obs	2 - p0, slope	7 - SEX, RACE, APOE, logAGE, logBMI, logHT, logWT	8	APOE - alpha, p0 logAGE - alpha, p0 logBMI - alpha logWT - p0	82	APOE - alpha, p0 logAGE - alpha, p0 logBMI - alpha logWT - p0	Identic	al	10.3
Lung cancer survival	Time-to-event 228 indiv - 165 events	2 - Te, <i>k</i>	5 - SEX, ecogPH, karnoPAT, karnoPH, age	13	AGE - k ecogPH - Te SEX - Te	36	AGE - k ecogPH - Te SEX - Te	identica	ıl	2.8



# **Performance of COSSAC**

• For the large majority of cases, the final covariate model is identical (11 out of 17) or very similar (4 out of 17) with COSSAC and SCM

	Number of models	Percentage
Identical models	11	64%
COSSAC model slightly better ( $\Delta$ LL < 3.84)	2	12%
SCM model slightly better ( $\Delta$ LL < 3.84)	2	12%
COSSAC model significantly better	1 (ΔLL = 40)	6%
SCM model significantly better	1 (ΔLL = 8.4)	6%

- COSSAC requires 2x 20x fewer runs
- Makes covariate search possible for big models that are intractable with SCM





#### **SAMBA**

#### Stochastic Approximation for Model Building Algorithm

Prague, M. & Lavielle, M. SAMBA: A novel method for fast automatic model building in nonlinear mixed-effects models. *CPT Pharmacometrics Syst. Pharmacol.* **11**, 161–172 (2022).

#### Add several covariates at once to be even faster.



#### Add several covariates at once to be even faster

#### First idea: add all covariates having a low p-value

Pearson's correlation test and/or ANOVA

	eta_	Tlag			eta	_ka	
	COEFF	STATISTICS	P-VALUE		COEFF	STATISTICS	P-VALUE
sex		1.14	2.95e-1	sex		0.036	8.5e-1
age	0.075	0.41	6.84e-1	age	0.0043	0.024	9.81e-1
wt	0.0013	0.0074	9.94e-1	wt	0.24	1.33	1.94e-1
	eta	a V			eta	Cl	
in .	COFFE	STATISTICS	P-VALUE	in the second se	COFFE	STATISTICS	P-VALUE
<b>L</b> -	COLIT	STATISTICS		-	COLIT	STATISTICS	TALOL
sex		18.59	1.61e-4	sex		0.38	5.43e-1
age	-0.039	-0.21	8.32e-1	age	0.32	1.83	7.78e-2
wt	0.75	6.24	7.22e-7	wt	0.35	2.08	4.65e-2

#### Add several covariates at once to be even faster

#### **First idea:** add all covariates having a low p-value

#### **Problem:**

some covariates might be correlated with each other and carry redundant information e.g weight and BMI





#### Add several covariates at once to be even faster

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#### **Problem:**

add all covariates s having a low p-value c

First idea:

V

some covariates might be correlated with each other and carry redundant information e.g weight and BMI

#### **Solution:**

For each parameter,

- linear regression between individual parameters and combinations of covariates
- Add covariates used in the best regression model (based on a BIC)

lowest BIC among regression models for V

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CRITERIA			COVARIALES							
	CRITERIA	AGE	SEX	WT			1.			
MODEL 1	-21.33			~	APPLY	$\log(V) = \alpha_0 + \beta_1 \times WT$	lo			
MODEL 2	-19.87		~	~	APPLY	$\log(V) = \alpha_0 + \beta_1 \times WT + \beta_2 \times SEX$	m			
MODEL 3	-17.88	×		~	APPLY	$\log(V) = \alpha_0 + \beta_1 \times WT + \beta_2 \times AGE$				
MODEL 4	-11.58		~		APPLY	$\log(V) = \alpha_0 + \beta_1 \times SEX$				
MODEL 5	-0.98					$\log(V) = \alpha_0$				
MODEL 6	2.44	×			APPLY	$\log(V) = \alpha_0 + \beta_1 \times AGE$				

#### **SAMBA procedure: overview**

- For each parameter, try all possible linear regression models
- Add the covariates corresponding to the best (BIC) linear model and run
- Repeat if best linear model has changed, otherwise stop

Prague, M. & Lavielle, M. SAMBA: A novel method for fast automatic model building in nonlinear mixed-effects models. *CPT Pharmacometrics Syst. Pharmacol.* **11**, 161–172 (2022).



#### **SAMBA procedure example**

4 parameters: Tlag, ka, V, Cl and 3 covariates: Age, Weight, Sex





For each parameter, best linear regression between samples from conditional distribution and covariates



#### **SAMBA procedure example**





### **SAMBA: comparison on real datasets**

		SCM		COSSAC	COSSAC			ΔBICc	
Dataset	Characteristics	#Runs <sup>b</sup>	Final Model <sup>a</sup>	#Runs <sup>b</sup>	Final Model 1	#Runs <sup>b</sup>	Final Model <sup>a</sup>	SAMBA-SCM	SAMBA-COSSAC
Warfarin	32 ind 247 obs.	44	logtWt - V, Cl	4	Identical	2	Identical	0	0
Linear PK	4 param 3 cov.		logtAge - C						
	4 re - 1 outcome								
Remifentanil	65 ind 1992 obs.	295	$\log LBM - V1$	13	logLBM - V1, V2	4	logLBM - V1	0.8	0.5
Linear PK	6 param 6 cov.		logAGE - <i>Cl</i> , Q2, Q3, V2, V3		logAGE - <i>Cl</i> , Q2, <i>V</i> 2, <i>V</i> 3		logAGE - <i>Cl</i> , <i>Q</i> 2, <b>Q</b> 3, <i>V</i> 2, <i>V</i> 3		
	4 re - 1 outcome		logBSA - Cl		logBSA - Cl		logBSA - Cl		
			$\log$ HT - $V2$						
					SEX - <b>V</b> 3		SEX - <b>V</b> 2		
Theophylline	12 ind 20 obs.	12	logtWEIGHT - ka	4	Identical	2	Identical	0	0
Linear PK	3 param 2 cov.								
	4 re - 1 outcome								
Quinidine	136 ind 361 obs.	22	none	11	Identical	1	Identical	0	0
Sparse PK	3 param 2 cov.								
	3 re - 1 outcome								
Tobramycin	97 ind 322 obs.	22	logCLCR - Cl	6	logCLCR - Cl	2	logCLCR - Cl	4.2	4.2
Sparse PK	3 param 2 cov.		$\log WT$ - $V$		$\log WT$ - $V$		logWT - Cl		
	2 re - 1 outcome								
Theophylline	18 ind 362 obs.	98	$\log WT$ - $Tlag1, V$	8	logWT - <b>Tlag</b> 1	6	$\log WT$ - $F, V$	-11.7	-27
Ext. Rel.	7 param 3 cov.				logAGE -ka2		logAGE -F		
Linear PK	7 re - 1 outcome					ka1,	logHT ka2, Tlag1, diffTlag2		

Prague, M. & Lavielle, CPT Pharmacometrics Syst. Pharmacol (2022).



## **SAMBA: comparison on real datasets**

		SCM	COS		OSSAC			ΔBICc	
Dataset	Characteristics	#Runs <sup>b</sup>	Final Model <sup>a</sup>	#Runs <sup>b</sup>	Final Model 1	#Runs <sup>b</sup>	Final Model <sup>a</sup>	SAMBA-SCM	SAMBA-COSSAC
Warfarin	32 ind 247+232 obs.	92	logWT - Cl	10	logWT - Cl	2	$\log WT - Cl, V$	-1.4	-1.4
PK/PD	8 param 3 cov.						logAGE -Cl, R0		
Joint	8 re - 2 outcomes								
Cholesterol	200 ind 1044 obs.	12	logAGE - Chol0, slope	5	logAGE - Chol0, slope	2	logAGE - Chol0	13.5	13.5
Disease	2 param 2 cov.		SEX - slope		SEX - slope				
Progression	2 re - 1 outcome								
Alzheimer	896 ind 3707 obs.	73	APOE - alpha, p0	8	APOE - alpha, p0	2	APOE - alpha, p0	6	1.5
Sparse PK	2 param 7 cov.		logAGE - <i>p</i> 0, alpha		logAGE - <i>p</i> 0, <b>alpha</b>		logAGE - p0		
	2 re - 1 outcome		logBMI - alpha		logBMI -alpha				
			logWT - <i>p</i> 0		logWT - p0		logWT - p0		
Tranexamic	166 ind 817 obs.	298	GROUP - Cl, V2	12	Identical	2	Identical	0	0
РК	4 param 10 cov.		logBMI - Cl						
	4 re - 1 outcome		logCOCK - Cl						
			logLBW - Q						
			logWeight - V2						

Prague, M. & Lavielle, CPT Pharmacometrics Syst. Pharmacol (2022). NASDAQ: SLP



# **Performance of SAMBA**

- For the majority of cases (7 out of 10), the final covariate model is identical (4 cases) or very similar (3 cases) with SAMBA and SCM
- SAMBA requires only very few runs (usually 2-3, max 6)
- Makes covariate search possible for big models that are intractable with SCM or COSSAC



#### SAMBA and COSSAC are available in Monolix



S**+** SimulationsPlus

 $\times$ 

wt

#### For versions >= 2019R1

# **Publications using COSSAC**

- COSSAC is available in Monolix since 2019, and article was published in 2021
- Method was used and cited in 22 scientific publications since then
- A few examples:
  - Idorsia: Krause, A., Lott, D., Brussee, J. M., Muehlan, C., & Dingemanse, J. (2023). Population pharmacokinetic modeling of daridorexant, a novel dual orexin receptor antagonist. CPT: Pharmacometrics & Systems Pharmacology, 12(1), 74–86.
  - Sanofi: Thai, H.-T., Gaudel, N., Cerou, M., Ayral, G., Fau, J.-B., Sebastien, B., van de Velde, H., Semiond, D., & Veyrat-Follet, C. (2022). Joint modelling and simulation of M-protein dynamics and progression-free survival for alternative isatuximab dosing with pomalidomide/dexamethasone. British Journal of Clinical Pharmacology, 88(5), 2052–2064.
  - BMS: Cheng, Y. et al. Model-based analysis for the population pharmacokinetics of **iberdomide** and its major active metabolite in healthy subjects and patients with relapsed and refractory multiple myeloma. Br. J. Clin. Pharmacol. 89, 316–329 (2023).
  - **GSK:** Yang, S., Simeoni, M. & Beerahee, M. Longitudinal Model-Based **Meta-Analysis** of Lung Function Response to Support Phase III Study Design in Chinese Patients With **Asthma**. Clin. Pharmacol. Ther. 111, 1286–1295 (2022).
  - Academics: Suñer, C. et al. Viral dynamics in patients with monkeypox infection: a prospective cohort study in Spain. Lancet Infect. Dis. 23, 445–453 (2023).



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Géraldine Cellière, Jonathan Chauvin, Jean-François Si Abdallah (Simulations Plus)

Key ideas of COSSAC and SAMBA Implementation of COSSAC in Rsmlx Optimization of COSSAC and SAMBA in Monolix



# References

#### Sampling from the conditional distribution

Lavielle, M. & Ribba, B. Enhanced Method for Diagnosing Pharmacometric Models: Random Sampling from Conditional Distributions. *Pharm. Res.* (2016)

#### COSSAC:

Ayral (Cellière), G., Si Abdallah, J. F., Magnard, C. & Chauvin, J. A novel method based on unbiased correlations tests for covariate selection in nonlinear mixed effects models: The COSSAC approach. *CPT Pharmacometrics Syst. Pharmacol.* **10**, 318–329 (2021).

#### • SAMBA:

Prague, M. & Lavielle, M. SAMBA: A novel method for fast automatic model building in nonlinear mixed-effects models. *CPT Pharmacometrics Syst. Pharmacol.* **11**, 161–172 (2022).





# Additional slides in case of questions

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# **COSSAC** detailed procedure





# **Availability in Monolix: scripts/batch**

#### In R via lixoftConnectors package

- calls the same C++ code than the GUI
- runModelBuilding(covariates= ..., parameters=..., strategy=..., criterion=..., relationships=..., threshold=...)

#### In R via the Rsmlx package

- beta implementation, slightly different from the final one in Monolix
- source code is open
- covariateSearch(project=..., method=..., covToTest=..., paramToUse=..., testRelations=..., settings=...)

#### In command line

monolix.bat --no-gui -t modelBuilding -s cossac -a 0.06 -r 0.001 -p "C:\Users\celliere\lixoft\monolix\demos\example.mlxtran"



#### **Performance of SAMBA in a simulation study**

**TABLE 2**Performance of the SAMBA algorithm for theselection of the covariate model in a simulation study using a one-compartment PK model

	Rsm	lx		Monolix				
Covariates	ka	V	СІ	ka	V	СІ		
$C_1$	2	100	100	2	100	100		
<i>C</i> <sub>2</sub>	0	1	100	0	1	100		
<i>C</i> <sub>3</sub>	1	2	1	2	2	1		
$C_4$	0	3	4	0	3	4		
$C_5$	0	1	1	1	2	1		

One hundred datasets of 100 individuals with 11 observations each have been generated. True model  $\mathcal{M}^*$  includes an effect of  $C_1$  on V and Cl and an effect of  $C_2$  on Cl. The percentages of times (over 100 replicates) each covariate-parameter relationship is selected in the final model are displayed. Implementation of SAMBA in *Rsmlx* and Monolix are compared.



# **Automatic model building: SAMBA**

SAMBA can work on

- Covariates
- Correlations between random effects
- Error models





# **Calculating number of runs**

The reduction in the number of runs between SCM and COSSAC will depend on the number of parameters p, the number of covariates c and the number of parameter-covariate relationships r in the final model.

In a SCM procedure, the first forward step requires  $p \times c$  runs, the second  $(p \times c) - 1$  runs, etc, until r relationships have been added. In the final forward step,  $(p \times c) - r$  runs are done and none is accepted. In the backward step(s), the r relationships can be tested for removal, corresponding to r runs. There is in general only one backward step. We thus obtain the following approximate formula:

$$\#\text{runs}_{\text{SCM}} \approx \left(\sum_{i=0}^{r} p \times c - i\right) + r$$
$$\#\text{runs}_{\text{SCM}} \approx p \times c \times (r+1) - r \times (r+1)/2 + r$$

With a COSSAC procedure, most often the first forward steps lead to an accepted model until the final model is reached. This corresponds to r forward runs approximately. They are then followed by forward not accepted models. Once models tend not be accepted anymore, there remain  $(p \times c) - r$  models which can be tested for additions, corresponding to at maximum  $(p \times c) - r$  runs. In practice only a fraction has a sufficiently low p-values to be run and evaluated. The backward steps are mostly models which have already run and do not require a new run, but we will consider that r relationships can be tested for removal. This leads to the approximate formula:

$$\#\text{runs}_{\text{COSSAC}} \approx r + (p \times c) - r + r = r + (p \times c)$$

## **COSSAC vs SCM: different final models**

For one run (remifentanil seqPD), the SCM method finds a model with one additional relationship (logAGE on E0) compared to COSSAC, which leads to an 8.4 points better LL. This better model is not tested by COSSAC because logAGE on E0 improves the LL only once covariates have been added on the gamma parameter, which has no variability and is tested after all others only. Running COSSAC again at the end would resolve the discrepancy but comes at a substantial cost in terms of runs. A similar situation happens for the Verapamil PK example.

On the opposite, for the model-informed drug development (MIDD) dataset, COSSAC finds a model that is 40 points better than SCM. The path of accepted runs taken by both methods is the same for the four first covariate additions. For the fifth, SCM adds nDiseases on the first order degradation rate  $(k_{deg})$  (largest LL decrease) and no further addition leads to a sufficient LL improvement. COSSAC adds logWT on Clr (smallest correlation p value). The LL improvement of this addition is smaller than that of nDiseases on  $k_{deg}$ , but this turns to be an advantage afterward, as the end-stage renal disease (ESRD) on Cl and logALB on renal clearance (Clr) can be added as additional significant covariates.

